Axion emission by magnetic-field induced conversion of longitudinal plasmons

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Magnetic fields mix axions with photons, allowing for the cyclotron process $e^- \to e^- a$ by virtue of an intermediate plasmon even if axions do not couple to electrons at the tree level. The axion and longitudinal-plasmon dispersion relations always cross for a certain wave number, leading to a resonant enhancement of this process. Even then, however, it cannot quite compete with the usual nucleon processes in a supernova core. The well-known axion window 10^{-5} eV$\lesssim m_a \lesssim 10^{-2}$ eV remains open and axions may still constitute the cosmic dark matter. [S0556-2821(98)03117-8]

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I. INTRODUCTION

Twenty years after their being postulated, axions still remain the most elegant solution of the strong *CP* problem [1,2], but alas, they also remain undiscovered. In terms of their mass, which almost uniquely characterizes a given axion model, the well-known ''window of opportunity'' 10^{-5} eV$\lesssim m_a \lesssim 10^{-2}$ eV has been left open by laboratory searches and by astrophysical and cosmological arguments [3–5]. Somewhere in this range axions could well be the main component or a significant fraction of cosmic dark matter. The ongoing search experiments for galactic dark-matter axions [6,7] offer our best chance to discover the existence of these ''invisible'' particles.

Meanwhile it remains of great interest to look for other astrophysical sites or laboratory experiments where the existence of axions could manifest itself. In the astrophysical context, relatively little attention has been paid to axionic processes in strong magnetic fields even though field strengths far in excess of the electron's Schwinger value of $B_e = m_e^2/e \simeq 4.41\times 10^{13}$ G may well exist in the interior of supernova cores or pulsars. Possible mechanisms to generate fields as strong as 10^{15}–10^{17} G in stars [8,9] or the early universe [10,11] are now widely discussed. Magnetic fields enable processes which are otherwise forbidden. An example is the cyclotron emission of axions $e^- \to e^- a$ in neutron stars or magnetic white dwarfs [12], even though the field strengths required to obtain an observable effect seem unrealistically large.

We here study a related process which is induced by a strong magnetic field, the axion cyclotron emission $e^- \to e^- a$ via a plasmon intermediate state. Therefore, the axion coupling is to the electromagnetic field so that our process does not depend on a direct axion-electron coupling which exists only in a restricted class of models [13]. The main motivation why our process could be expected to be non-negligible is that it is resonant at a particular energy of the emitted axion, provided that the intermediate plasmon is longitudinal, because the axion and the longitudinal plasmon dispersion relations always cross for a certain wave number (Fig. 1).

Our final result will be that even for the largest magnetic fields that may plausibly exist in supernova cores, this new emission process is less important than the usual nucleon bremsstrahlung rate. Therefore, the axion ''window of opportunity'' remains unaffected by our process. While from a practical astrophysical perspective our results turn out to be purely academic, we still find it worthwhile to communicate this calculation for its conceptual value.

In Sec. II we calculate the emission rate when our effect is pictured as a cyclotron process $e^- \to e^- a$. In Sec. III we calculate it again, picturing it as a plasmon-axion oscillation phenomenon. Because we are on resonance, both calculations yield the same result. In Sec. IV we summarize our findings.

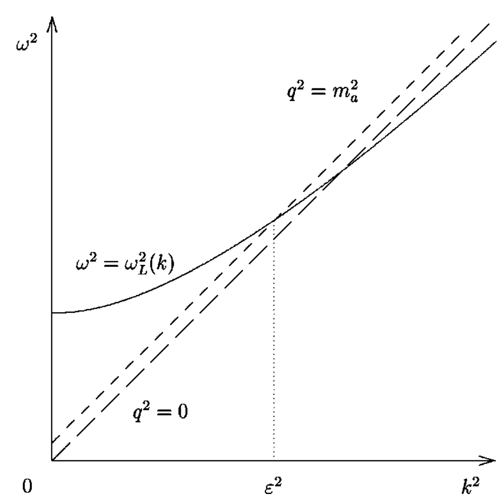

FIG. 1. Dispersion relation $\omega^2 = \omega_L^2(k)$ for longitudinal plasmons (solid line), axions $\omega^2 = k^2 + m_a^2$ (short dashed line), and vacuum photons $\omega = k$ (long dashed line).

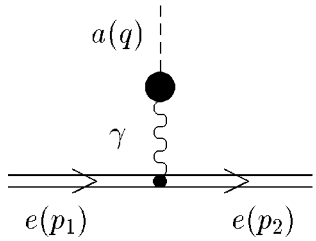

FIG. 2. Cyclotron emission process.

II. CYCLOTRON PROCESS

A. Axion-photon coupling

The cyclotron process $e^- \to e^- a$ via an intermediate plasmon is shown in Fig. 2, where double lines correspond to the electron wave functions in the magnetic field. The effective Lagrangian describing the axion-photon coupling in the presence of an external field can be expressed as

$$\mathcal{L}_{a\gamma} = \overline{g}_{a\gamma} \partial_\mu A_\nu \tilde{F}^{\mu\nu} a, \tag{1}$$

where A is the four-potential of the quantized electromagnetic field, $\tilde{F}$ is the dual of the tensor which describes the external field, and a is the axion field.

The effective a-γ-coupling constant with dimension $(\text{energy})^{-1}$ is

$$\overline{g}_{a\gamma} = g_{a\gamma} + \frac{\alpha}{\pi} \sum_{\text{light } f} \frac{Q_f^2 g_{af}}{m_f}. \tag{2}$$

Here $g_{a\gamma}$ is the usual coupling constant in a vacuum [4], $g_{a\gamma} = \alpha\xi/2\pi f_a$, where ξ is a model-dependent parameter and f_a is the Peccei-Quinn scale. Further, $g_{af} = C_f m_f / f_a$ is a dimensionless Yukawa coupling constant of axions to both quarks and leptons at the tree level with C_f a model-dependent factor, m_f the fermion mass, and Q_f its relative electric charge. The second term in Eq. (2) is the field-induced part of the coupling which derives from the diagram of Fig. 3. It has contributions from light fermions f for which $\chi_f \gg 1$. The dynamic parameter is defined as

$$\chi_f^2 = \frac{e_f^2 (qFFq)}{m_f^6} \tag{3}$$

with $e_f = Q_f e$ the fermion electric charge and q the axion four-momentum.

The effective axion-photon coupling constant in the presence of an external field is then written as

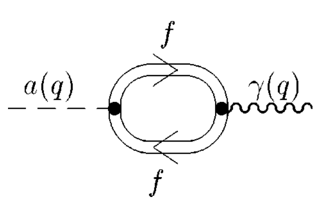

FIG. 3. Magnetic-field induced modification of the axion-photon coupling.

$$\overline{g}_{a\gamma} = \frac{\alpha}{2\pi f_a} \left(\xi + 2 \sum_{\text{light } f} Q_f^2 C_f \right). \tag{4}$$

Note that for ultra-relativistic electrons ($E \gg m_e$) the case $E^2 \gg eB$, where a large number of the Landau levels are excited, is well described by the crossed-field limit $(e^2 pFFp)^2 \gg (e^2 FF)^3$.

B. Matrix element

The matrix element of the $e^- \to e^- a$ decay, corresponding to the diagram of Fig. 2, can be written as

$$S = \frac{\overline{g}_{a\gamma}}{\sqrt{2E_a V}} hJ \tag{5}$$

in terms of the currents

$$J_\alpha = \int d^4x \, \overline{\psi}(p_2, x) \gamma_\alpha \psi(p_1, x) e^{iqx},$$

$$h_\alpha = -ieq_\mu \tilde{F}^{\mu\nu} G^L_{\nu\alpha}(q). \tag{6}$$

Here, $e>0$ is the elementary charge, $q=(E_a, \mathbf{q})$ is the four-momentum of the emitted axion, and $p_1 = (E_1, \mathbf{p}_1)$ and $p_2 = (E_2, \mathbf{p}_2)$ are the quasi-momenta in the crossed field of the initial- and final-state electrons with $p_1^2 = p_2^2 = m_e^2$; in the zero-field limit $p_{1,2}$ become the free electron four-momenta.

Further, $\psi(p,x)$ is the solution of the Dirac equation in a constant crossed field $F_{\mu\nu} = k_\mu a_\nu - k_\nu a_\mu$ where $A_\mu = a_\mu \varphi$ is the four-potential with $\varphi = kx$ and $k^2 = ak = 0$. We have, explicitly,

$$\psi(p,x) = \left(1 - \frac{e\hat{k}\hat{a}}{2kp} \varphi \right) \frac{U(p)}{\sqrt{2EV}} e^{iI(p,x)}, \tag{7}$$

where $\hat{k} \equiv \gamma_\mu k_\mu$ and

$$I(p,x) = -px + \frac{eap}{2kp}\varphi^2 + \frac{e^2 a^2}{6kp}\varphi^3. \tag{8}$$

The bispinor $U(p)$, which is normalized by the condition $\overline{U}U = 2m_e$, satisfies the Dirac equation for the free electron $(\hat{p} - m_e)U(p) = 0$.

Finally, $G^L_{\alpha\beta}$ is the longitudinal plasmon propagator. Because the magnetic field is taken to be weak on the scale of the medium temperature T and electron Fermi momentum p_F ($T^2, p_F^2 \gg eB$), one can use the zero-field propagator

$$G^L_{\alpha\beta} = i \frac{l_\alpha l_\beta}{q^2 - \Pi^L}. \tag{9}$$

Here, l_α and Π^L are the eigenvector and eigenvalue of the polarization operator corresponding to the longitudinal plasmon, respectively, with

$$l_\alpha = \sqrt{\frac{q^2}{(uq)^2 - q^2}} \left(u_\alpha - \frac{uq}{q^2} q_\alpha \right) \tag{10}$$

and u_α the four-velocity of the medium.

The general expression for the matrix element, Eq. (5), is rather cumbersome. It is substantially simplified in the ultra-relativistic limit where $E_{1,2} \gg m_e$. After integration over the space-time coordinates x it is

$$S \simeq \frac{(2\pi)^4 \delta^2(\mathbf{Q}_\perp)\,\delta(\chi_1-\chi_2-\chi_q)}{\sqrt{2E_aV2E_1V2E_2V}} \times \frac{\overline{g}_{a\gamma}}{\pi m_e^2 r}\Phi(\eta)\overline{U}(p_2)\hat{h}U(p_1), \tag{11}$$

where $Q=p_1-p_2-q$ is the four-momentum transfer to the external field and $Q_\perp$ its perpendicular component defined by the condition $\mathbf{Q}_\perp \cdot \mathbf{k}=0$. Further, $r=(\chi_a/2\chi_1\chi_2)^{1/3}$, $\chi_i^2 = e^2(p_iFFp_i)/m_e^6$ for $i=1,2,a$ with $p_a=q$ the axion momentum, and $\eta=r^2(1+\tau^2)$ with $\tau=-e(p_1\tilde{F}q)/(m_e^4\chi_a)$. Finally,

$$\Phi(\eta)=\int_0^\infty dy\ \cos\left(\eta y+\frac{y^3}{3}\right) \tag{12}$$

is the Airy function.

C. Resonant transition rate

After integrating over the final-state electron momentum and the axion solid angle, the differential probability of $e^- \to e^- a$ is

$$\frac{dW}{dE_a} \simeq \frac{\overline{g}_{a\gamma}^2}{12\pi}(eB)^2\,\frac{E_a^2\cos^2\theta}{(\mathcal{E}^2-E_a^2)^2+\gamma^2\mathcal{E}^4}\left(1-\frac{E_a}{E_1}\right), \tag{13}$$

where θ is the angle between the external magnetic field $\mathbf{B}$ and the momentum $\mathbf{p}_1$ of the initial electron. $\mathcal{E}$ is the energy where the axion and longitudinal plasmon dispersion curves cross. In an ultra-relativistic degenerate plasma it is

$$\mathcal{E}^2 \simeq \frac{4\alpha}{\pi}\mu^2\left(\ln\frac{2\mu}{m_e}-1\right), \tag{14}$$

where μ is the electron chemical potential.

The dimensionless resonance width γ of the $e^-\to e^- a$ process in Eq. (13) is

$$\gamma=\frac{\mathcal{E}\Gamma_L(\mathcal{E})}{q^2 Z_L}, \tag{15}$$

where at the resonance point, of course, $q^2=m_a^2$. The plasmon wave-function renormalization factor is

$$Z_L^{-1}=1-\frac{\partial \Pi^{(L)}}{\partial q_0^2}. \tag{16}$$

In an ultra-relativistic degenerate plasma it is

$$Z_L q^2 \simeq 2\mathcal{E}^2\,\frac{m_e^2[\ln(2\mu/m_e)-1]}{\mu^2} \tag{17}$$

with the electron chemical potential μ.

Further, $\Gamma_L(\mathcal{E})$ is the total width of the longitudinal plasmon due to the processes $\gamma_L \to \nu\overline{\nu}$, $\gamma_L \to e^+e^-$, $\gamma_L e^- \to e^-$, and so forth in the presence of the magnetic field. It turns out that the main contribution comes from the inverse cyclotron process $\gamma_L e^- \to e^-$. A direct calculation of the absorption rate due to this process yields

$$\Gamma_{\rm ab}(\mathcal{E}) \simeq \frac{2\alpha}{3}q^2 Z_L\,\frac{\mu^2}{\mathcal{E}^3}\,\frac{e^{\mathcal{E}/T}}{e^{\mathcal{E}/T}-1}. \tag{18}$$

For the total plasmon width in the medium one has to take into account the longitudinal plasmon creation process with a probability $\Gamma_{\rm cr}(\mathcal{E})=e^{-\mathcal{E}/T}\Gamma_{\rm ab}(\mathcal{E})$ so that altogether [14]

$$\Gamma_L=\Gamma_{\rm ab}-\Gamma_{\rm cr}=(1-e^{-\mathcal{E}/T})\Gamma_{\rm ab}. \tag{19}$$

This "width" plays the role of the imaginary part of the polarization operator $\mathrm{Im}\,\Pi^L(\mathcal{E})=-\mathcal{E}\Gamma_L(\mathcal{E})$. With these results one finds, for Eq. (15),

$$\gamma=\frac{2\alpha}{3}\,\frac{\mu^2}{\mathcal{E}^2}. \tag{20}$$

The temperature dependence has magically canceled.

D. Axion emissivity

In order to obtain the plasma's axion emissivity we finally need to integrate over the initial-state electron phase space as well as the axion energies,

$$Q_a=\int \frac{2d^3\mathbf{p}_1}{(2\pi)^3}\int dE_a\frac{dW}{dE_a}E_a f(E_1)[1-f(E_2)], \tag{21}$$

where $E_2=E_1-E_a$ and $f(E)=(e^{(E-\mu)/T}+1)^{-1}$ is the electron's Fermi-Dirac distribution function. For a degenerate plasma with $\mu \gg T$ the integral over the initial electron energy E_1 can be easily calculated:

$$J=\int dE_1 f(E_1)[1-f(E_2)]R(E_1) \simeq \frac{E_a R(\mu)}{e^{E_a/T}-1}. \tag{22}$$

With this result the emissivity is

$$Q_a \simeq \frac{\mu^2}{2\pi^2}\int_0^\pi \sin\theta\, d\theta \int_0^\infty \frac{dE_a E_a^2}{e^{E_a/T}-1}\,\frac{dW}{dE_a}. \tag{23}$$

Taking into account Eqs. (13) and (20), the integration over the angle θ and the axion energy E_a gives

$$Q_a \simeq \frac{\overline{g}_{a\gamma}^2}{48\pi^2\alpha}(eB)^2\frac{\mathcal{E}^3}{e^{\mathcal{E}/T}-1}. \tag{24}$$

For a numerical evaluation note that the combination eB is independent of the system of electromagnetic units. In rationalized natural units we have $\alpha=e^2/4\pi \simeq 1/137$ and the field strength of 1 G corresponds to 1.95×10^{-2} eV^2.

Equation (24) gives us the emissivity of an ultra-relativistic degenerate plasma in the resonance region. When the plasma is strongly degenerate we may have $\mathcal{E} \gg T$. In this case the resonant emissivity is exponentially suppressed, and so the nonresonant contribution

$$Q_a \simeq \frac{2\zeta(5)}{3\pi^3} \overline{g}_{a\gamma}^2 (eB)^2 \frac{\mu^2 T^5}{\mathcal{E}^4} \tag{25}$$

will dominate.

III. PLASMON-AXION CONVERSION

We have calculated the cyclotron emission process in the resonance region. It is known that in this case $e^- \to e^- a$ effectively reduces to two consecutive processes: cyclotron radiation of a real longitudinal plasmon $e^- \to e^- \gamma_L$ and a subsequent plasmon-axion transition $\gamma_L \to a$. Therefore, one may simply picture the axion emission as a result of the plasmon-axion conversion in the presence of the magnetic field.

Starting from the effective Lagrangian, Eq. (1), the matrix element for the $\gamma_L(q) \to a(q')$ transition can be written as

$$S \simeq \frac{\overline{g}_{a\gamma}(2\pi)^4 \delta^4(q-q')}{\sqrt{2\omega V 2E_a V}} \sqrt{Z_L}(l\tilde{F}q), \tag{26}$$

where l_α is the four-vector of the longitudinal plasmon polarization defined in Eq. (9), $q=(\omega,\mathbf{q})$ is the initial-state plasmon four-momentum, and $q'=(E_a,\mathbf{q}')$ is the final-state axion four-momentum. The transition rate is

$$dW = \frac{\pi \overline{g}_{a\gamma}^2}{2\omega E_a} \delta^4(q-q') Z_L (l\tilde{F}q)^2 d^3\mathbf{q}', \tag{27}$$

leading to an axion emissivity of

$$Q_a = \int \frac{d^3\mathbf{q}}{(2\pi)^3} \int dW \frac{E_a}{e^{\omega/T}-1}. \tag{28}$$

Integrating the phase space of the initial plasmon and final axion this becomes

$$Q_a \simeq \frac{\overline{g}_{a\gamma}^2}{48\pi^2\alpha} \frac{(eB)^2 \mathcal{E}}{e^{\mathcal{E}/T}-1} \left(\frac{q^2 Z_L}{1-d\omega_L/dk} \right)_{k=\mathcal{E}}. \tag{29}$$

Here, $\omega_L(k)$ is the plasmon energy as a function of wave number (see Fig. 1). The expression in brackets is always found to be $\mathcal{E}^2$, independently of the plasma conditions (relativistic, nonrelativistic, degenerate, nondegenerate). Therefore, we immediately recover Eq. (24).

We conclude that the axion emissivity, Eq. (24), which we calculated for an ultra-relativistic degenerate plasma, *is actually valid in the general case*. We only need to identify the crossing point $\mathcal{E}$ of the dispersion relations for the relevant plasma conditions. For a nonrelativistic plasma we find

$$\mathcal{E}^2 \simeq 4\pi\alpha \frac{n_e}{m_e}, \tag{30}$$

where n_e is the electron density. For an ultra-relativistic nondegenerate plasma it is

$$\mathcal{E}^2 \simeq \frac{4\pi\alpha}{3} T^2 \left[\ln\left(\frac{4T}{m_e}\right) + \frac{1}{2} - \gamma_E + \frac{\zeta'(2)}{\zeta(2)} \right], \tag{31}$$

where Euler's constant γ_E is 0.577216 and $\zeta'(2)/\zeta(2) = -0.569961$ so that $1/2 - \gamma_E + \zeta'(2)/\zeta(2) = -0.647177$. For an ultra-relativistic degenerate plasma $\mathcal{E}^2$ is given by Eq. (14).

IV. DISCUSSION AND SUMMARY

We have calculated the axion emissivity of a plasma in the presence of a strong magnetic field due to the resonant transition between longitudinal plasmons and axions. The field was taken to be strong in the sense $eB \gg \alpha^3 E^2$, where E is a typical electron energy, so that phase space is opened for the cyclotron process $e^- \to e^- a$, and yet it was also taken to be weak in the sense $E^2 \gg eB$ so that the electron energy is still the largest energy scale in the problem. For the plasma dispersion relation we could then use the zero-field expressions. We have computed the emission rate as a cyclotron process $e^- \to e^- a$ with an intermediate plasmon on resonance and also directly as a transition of thermally excited plasmons into axions. Both calculations yield the same result, Eq. (24), in terms of the crossing energy $\mathcal{E}$ of the axion with the longitudinal plasmon dispersion relation. With the appropriate expression for $\mathcal{E}$ this result applies to all plasma conditions.

While this process and its evaluation are conceptually quite intriguing, the actual energy-loss rate is rather small. If we take the conditions in a supernova core after collapse as an example, an electron chemical potential of 200 MeV is a representative value, leading to $\mathcal{E} = 46$ MeV. With a temperature $T = 30$ MeV we thus have $\mathcal{E}/T = 1.53$. We express the axion-photon coupling as $g_{a\gamma} = m_{\rm eV}/0.69 \times 10^{10}$ GeV for typical axion models in terms of the axion mass with $m_{\rm eV} = m_a/1$ eV. The emission rate is then

$$Q_a = 2.0 \times 10^{30}\ \mathrm{erg\ cm^{-3}\ s^{-1}} \frac{m_{\rm eV}^2 B_{16}^2 \mu_{200}^3}{e^{\mathcal{E}/T}-1}, \tag{32}$$

where $B_{16} = B/10^{16}$ G and $\mu_{200} = \mu/200$ MeV. With $\mathcal{E}/T = 1.53$ the denominator is 3.62 so that $Q_a = 0.6 \times 10^{30}\ \mathrm{erg\ cm^{-3}\ s^{-1}}\ m_{\rm eV}^2 B_{16}^2$. Dividing by a typical mass density of $\rho = 3 \times 10^{14}\ \mathrm{g\ cm^{-3}}$ this is $Q_a/\rho = 2 \times 10^{15}\ \mathrm{erg\ g^{-1}\ s^{-1}}\ m_{\rm eV}^2 B_{16}^2$. This is to be compared with the neutrino energy loss rate of a supernova core during its first few seconds after collapse of around $10^{19}\ \mathrm{erg\ g^{-1}\ s^{-1}}$. Comparing this number with the zero-field emission rate by nucleon bremsstrahlung processes already gives a limit of around $m_a \lesssim 10^{-2}$ eV so that an unrealistically large magnetic field would be required to cause a significant plasmon-conversion rate.

We may turn this finding around and conclude that the axion ''window of opportunity'' remains open. Even huge magnetic fields in a supernova-core do not seem to change the usual picture that axions are emitted primarily by fluctuating nucleon spins.

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